647510 **MSC2MatE201x** **Seat No :________**

M.Sc.(Maths.) Semester - 2 (*CBCS*) Examination
March/April - 2024 (NEW COURSE)
Classical Mechanics -2 (Elective)

Time: 02:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que. 1 (A) Answer any One out of Two **[04]**

(1) Discuss and derive the formula for the moment of inertia about the axis of rotation.

(2) Define angular momentum, inertia tensor, moment of inertia and kinetic energy of motion.

(B) Answer any Two out of Three **[10]**

(1) Prove that the kinetic energy and the angular momentum are constants throughout the motion.

(2) Prove that the general motion of the top about the fixed point is the combination of nutation and precession.

(3) Discuss the stability of steady motion when the axis of the top is vertical.

Que. 2 (A) Answer any One out of Two **[04]**

(1) Discuss Lorentz transformations.

(2) Explain the basic program of special relativity and its significance in classical mechanics.

(B) Answer any Two out of Three **[10]**

(1) Derive Lorentz transformations in real four dimensional spaces.

(2) Analyze the differences between force equations in relativistic mechanics and classical mechanics, highlighting any key distinctions.

(3) Critically analyze the force and energy equations in relativistic mechanics, comparing them with classical counterparts and discussing their broader implications.

Que. 3 (A) Answer any One out of Two **[04]**

(1) Derive Hamilton's equations of motion from Hamilton's modified principle.

(2) Derive matrix form of Hamilton's equations of motion.

(B) Answer any Two out of Three **[10]**

(1) Derive Lagrange's equation of motion from Hamilton's equation of motion for a system of n degrees of freedom.

(2) Obtain Routhian equation of motion for a system with Lagrangian

$$L = \frac{m}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2\right) + \frac{k}{r}.$$

(3) Find the Hamiltonian and Hamilton's equation of motion for the Lagrangian

$L = \dot{q}_1^2 + \frac{\dot{q}_1^2}{a+bq_1^2} + k_1 q_1^2 + k_2 \dot{q}_1 \dot{q}_2$, where q_1, q_2 are generalized coordinates.

Que. 4 (A) Answer any One out of Two **[04]**

(1) State and prove Poisson's theorem.

(2) (a) Prove that $[uv, w] = u[v, w] + v[u, w]$.

(b) Show that Poisson bracket is bilinear.

(B) Answer any Two out of Three **[10]**

(1) Discuss Hamilton's equations of motion in Poisson's bracket.

(2) Derive canonical transformation generated by $F_1(q, Q, t)$ and $F_2(q, P, t)$.

(3) Prove that (a) $[u, v] = -[v, u]$ (b) $[u + v, w] = [u, w] + [v, w]$.

Que. 5 **(A)** **Answer any One out of Two** **[04]**

(1) Discuss Hamilton-Jacobi equation for Hamilton's principal function.

(2) If a rectangular parallelepiped with its edges $2a, 2a, 2b$ rotates about its Centre of gravity under no forces. Prove that its angular velocity about one principal axis is constant.

(B) **Answer any Two out of Three** **[10]**

(1) If u, v, w are functions of q and p only then prove that

$$\left[u, [v, w]\right] + \left[v, [w, u]\right] + \left[w, [u, v]\right] = 0.$$

(2) Using cylindrical coordinates (ρ, ϕ, z), write the Hamilton's equations of motion for a particle of mass m moving in a force field of potential $V(\rho, \phi, z)$.

(3) Show that Poisson's bracket is invariant under canonical transformation.

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